**MA 125 6D, CALCULUS I**
October 14, 2015

Name (Print last name first): ........................................

**Show all your work and justify your answer!**

**No partial credit will be given for the answer only!**

**PART I**

**You must simplify your answer when possible.**
**All problems in Part I are 10 points each.**

1. Find the derivative of the function $y = f(x) = \sin(x^3)$.

$f'(x) = \cos(x^3) \cdot 3x^2$

2. Find the derivative of $f(x) = (x^2 + x + 1)^6$.

$f'(x) = 6\,(x^2+x+1)^5\,(2x+1)$

3. Find the absolute maximum and minimum of the function
$y = f(x) = (3x-4)^2(x+1)^2$ on the interval $[0,1]$.

① $f'(x) = 2(3x-4)\,3\,(x+1)^2 + (3x-4)^2 \cdot 2 \cdot (x+1)$

$= (3x-4)(x+1)\{6(x+1) + 2(3x-4)\}$

$= (3x-4)(x+1)(6x+6+6x-8)$

$= (3x-4)(x+1)(12x-2) = 0$ $\Rightarrow f(\frac{1}{6}) = (\frac{1}{2}-4)^2(\frac{1}{6}+1)^2$

16.8

$\Rightarrow$ critical point $x = \frac{1}{6}$, $\frac{4}{3}, -1$ not in $[0,1]$

② $f(0) = (-4)^2(1)^2 = 16$, $f(1) = (3-4)^2(1+1)^2 = 4$

③ ~~local m~~ Abs max: 16.8  Abs min: 4. by closed interval method.

4. Find the linearization of the function $f(x) = 3\sqrt[3]{x+1}$ at the point $a = 7$ and use it to estimate the value $f(6.9)$.

$L(x) = f'(7)(x-7) + f(7)$

$= \frac{1}{4}(x-7) + 6$

$f'(x) = (x+1)^{-\frac{2}{3}}$

$\Rightarrow f'(7) = 8^{-\frac{2}{3}} = \frac{1}{4}$

$f(7) = 6$

$\Rightarrow f(6.9) \approx L(6.9)$

$= \frac{1}{4}(6.9-7) + 6$

$= \frac{1}{4}(-0.1) + 6$

$= 6 - 0.025 = 5.975.$

5. Find two positive numbers so that their sum is 10 and their product is maximal. [As always you must justify your answer!]

$x+y=10$, $f(x,y)=xy$ maximum?

$\Rightarrow f(x) = x(10-x) = 10x - x^2$

$\Rightarrow f'(x) = 10 - 2x = 0 \Rightarrow \boxed{x=5}$

$\Rightarrow x=5$, $y=5$ then ~~f~~

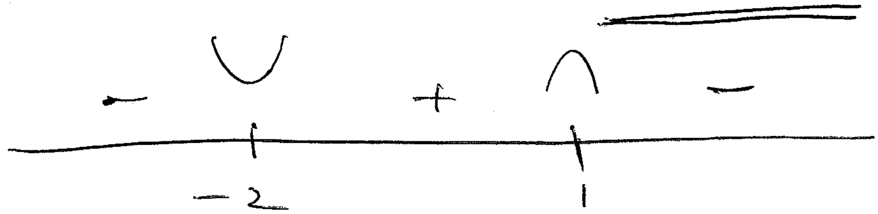
their product is $\underline{25}$ maximum.

6. Suppose that the **derivative** of a function $y = f(x)$ is given:

$f'(x) = (x+2)(1-x)$.

(a) Find the $x$-coordinates of all local max/min of the function $y = f(x)$.

$f'(x) = (x+2)(1-x) = 0 \Rightarrow \underline{x = -2, 1}$




local mini    local max

x coordinate is −2 ,    : 1

(b) At which $x$ value is the function $y = f(x)$ most rapidly increasing?

$f''(x) = (1-x) - (x+2) = 1 - x - x - 2$

$= -2x - 1 = 0 \Rightarrow \boxed{x = -\frac{1}{2}}$

## PART II

7. **[15 points]** A store has been selling 100 speakers a week at \$200 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of units sold will increase by 10 a week. Find the demand function and the revenue function. What large a rebate should the store offer to maximize its revenue?

   You may use the relation between the revenue $R(x)$ and demand function $p(x)$, $R(x) = xp(x)$ and you should find demand function firstly.

$$p(x) = 200 - \frac{10}{10}(\cancel{200}\, x - 100)$$

$$= -x + 300$$

$$\Rightarrow R(x) = x \cdot p(x) = -x^2 + 300x$$

$$\Rightarrow R'(x) = -2x + 300 = 0$$

$$\Rightarrow \boxed{x = 150}$$

R(150).

$$\Rightarrow p(150) = \cancel{200} - 150 + 300 = \underline{150}.$$

$200-150$

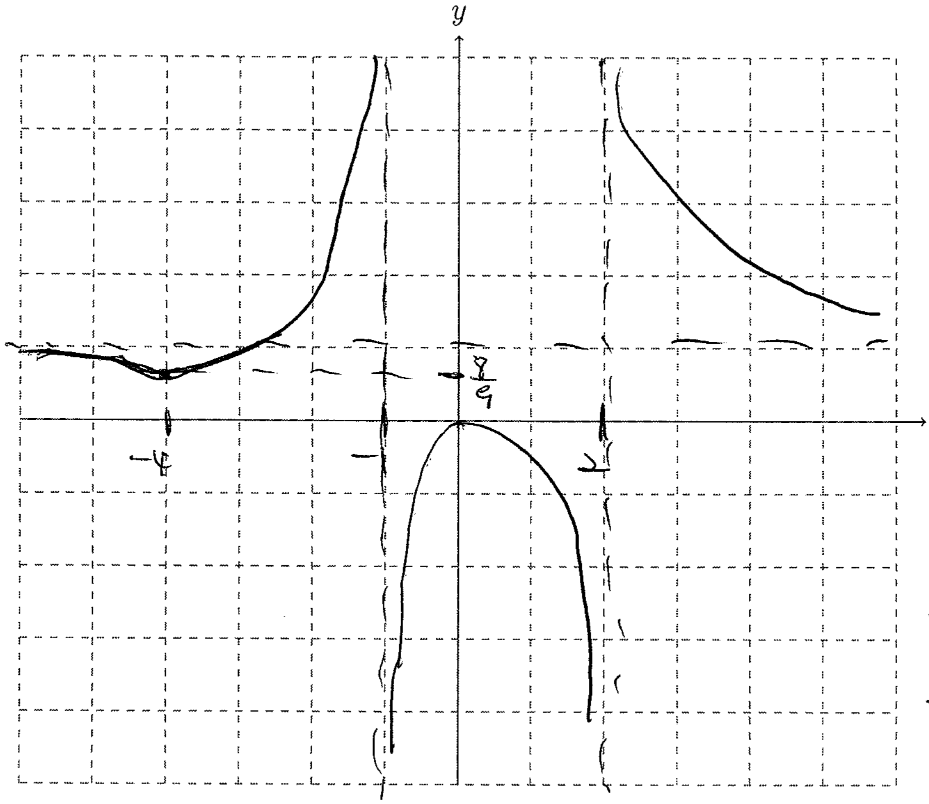
$\Rightarrow$ We conclude that when the rebate 50\$, we can have maximum revenue and can sell 150 speakers a week.

8. [**20 points**] **Use calculus** to graph the function $y = f(x) = \dfrac{x^2}{x^2 - x - 2}$. Indicate

- $x$ and $y$ intercepts,
- vertical and horizontal asymptotes (if any),
- in/de-creasing; local/absolute max/min (if any).

You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).

② horizontal asymptote $y = 1$




$-\ \vee\ +\ \wedge\ -$ (at $-4$, $0$)

local min | local max

$f(-4) = \dfrac{16}{18} = \dfrac{8}{9}$ | $f(0) = 0$

① $x$-intercept : 0 , $y$ intercept : 0

② vertical asymptotes $x^2 - x - 2 = (x+1)(x-2) = 0 \Rightarrow x = -1$, $x = 2$

and $\lim_{x\to -1^-} f(x) = +\infty$ $\lim_{x\to -1^+} f(x) = -\infty$

$\lim_{x\to 2^-} f(x) = -\infty$ $\lim_{x\to 2^-} f(x) = +\infty$

③ $f'(x) = \dfrac{2x(x^2-x-2) - x^2(2x-1)}{(x^2-x-2)^2} = \dfrac{-x(x+4)}{(x^2-x-2)} \Rightarrow$ critical point $x = 0, -4$

9. **[5 points]** Find the number $c$ that satisfies the conclusion of the Mean Value Theorem on the given interval. $f(x) = \sqrt{x^3}$, $[0, 1]$.

① $f'(x) = (x^{\frac{3}{2}})' = \frac{3}{2} x^{\frac{1}{2}}$.

$\Rightarrow f'(c) = \frac{3}{2} c^{\frac{1}{2}}$.

② $\frac{f(b) - f(a)}{b - a} = \frac{f(1) - f(0)}{1 - 0}$

$= \frac{1 - 0}{1 - 0} = 1$.

$\Rightarrow \frac{3}{2} c^{\frac{1}{2}} = 1 \Rightarrow c^{\frac{1}{2}} = \frac{2}{3}$ $\therefore \boxed{c = \frac{4}{9}}$